1 Problem Description and Biological Background

Male and female moths communicate in complex ways, as they optimize their chances of finding and attracting the best mate they can. Female moths attract mates by emitting small amounts of pheromones that males can detect with their elaborately shaped antennae. Competition is fierce: many males aim to reach the female and the “winner” must reach her before everybody else. The chemoattractant must be a reliable guide for directing motion through winds and obstacles, and even low quantities of chemoattractants must be detected throughout the journey. What strategies should males use to find a mate?

With a lifetime of 7 days, there is a sense of urgency for the female moths to find a mate. What strategies for emitting pheromones should female moths employ to maximize their encounters with the fittest mates? For example, females may emit pheromones more frequently to increase the chance they find a mate, albeit a less fit one. One measure of the fitness of the mate is the antenna quality—only the fittest individuals will have the largest and most intricate antennae as it is costly to maintain and produce such structures.

To date, very little work has been done on understanding the complex interplay between moths at the mating stage. The aim of this project is to create, de novo, an agent-based model where the female moth emits a diffusing chemoattractant and multiple male moths respond to it, trying to detect her, within a finite time. The quantity of interest is the likelihood that a “good” male will mate with the female.

2 Male Moth Movements

We assume that, before detection of pheromone emitted by a female moth, male moths fly randomly. However, once male moths detect pheromone, they exploit the fact that the pheromone must have been advected from a location upwind of their present position and then orient and “surge” upwind. Turbulence of the ambient air makes pheromone plumes intermittent and filamentous, and thus male moths may lose contact with plume as they surge upwind. Upon loss of pheromonal signal, males moths adopt a zig-zagging trajectory perpendicular to the direction of wind flow. This “casting” behavior perpendicular to the wind increases the probability that a male moth will reacquire a pheromone filament that has been advected away by turbulent air currents. After casting unsuccessfully for a period of time, a male moth will resume random flight.

In our agent-based model of male moth movement, we assume each male moth moves at the same constant speed and the wind is in the positive horizontal direction. Male moths take one of four possible behavior states that determine the direction of their movement as shown below.
(a) **Diffusion:** Male moths choose a direction drawn uniformly from \((0, 2\pi)\). Wind direction is shown as gray arrows.

(b) **Surging:** While detecting pheromone signal, male moths choose a direction drawn uniformly from \((\pi - \delta, \pi + \delta)\) for some \(\delta\).

(c) **Casting:** Upon loss of pheromone signal, male moths choose a direction drawn uniformly from \((\pm \frac{\pi}{2} - \delta, \pm \frac{\pi}{2} + \delta)\) for some \(\delta\).

(d) **Find female and mate!**: When a male moth is within a certain radius of a female moth, it is assumed that they find love and mate.

### 3 Pheromone Plume

To model the pheromone plume that is released by the female, we implement a Gaussian plume model, a standard approach for studying the transport of airborne contaminants due to turbulent diffusion and advection by the wind. A sample simulation of the plume and male moths is shown to the right.

This plume is based on a constant atmospheric diffusivity \(D\) and steady wind velocity \(u > 0\) in the positive \(x\) direction. For an instantaneous emission of chemoattractant of strength \(Q\) from the origin, the downfield concentration field is modeled by the Gaussian puff solution

\[
c(x, y, t) = \frac{Qu}{4\pi Dx} \exp \left[ -u \left( \frac{(x - ut)^2 + y^2}{4Dx} \right) \right].
\]

In our simulations, we construct time-dependent female pheromone broadcasting strategies by superimposing multiple Gaussian puff solutions.

### 4 Female Moth Pheromone Broadcasting Strategies

Female moths attract mates by emitting small amounts of pheromones that males detect with their elaborately shaped antennae. We examined various pheromone emitting strategies over the course of 2 hours, effectively modeling the behavior of the female moths during one evening. Some of the strategies we examined are below, showing the amount of pheromone released over time.
5 Discussion and Future Directions

We had progress in two of the project goals proposed by the mentors, in particular, constructing a two-dimensional agent-based model of a moth cohort and identifying model parameters and their physically-relevant values. We examined the situation where the female pheromone plume was fixed in time and the male moths were randomly assigned different wind and pheromone sensing abilities. Preliminary results showed that male moths that were good at both wind sensing and pheromone sensing were the most successful at mating. We plan to continue our efforts in these two areas to validate and strengthen our model as well as to further examine the effect of male moth abilities on probability of mating success.

We plan to do a parameter sweep to look at the trade-off between number of male moths that make it to a female moth depending on the strategies that they employ. What strategy results in the most moths mating in the shortest time? We also plan to add more female moths in the simulation and add a repulsive term between male moths in the model, as our current model effectively simulates 1 female moth and many independent simulations of 1 male moth.

Two of the other project goals proposed by the mentors we have yet to target but plan to do so in the following months. One involves analyzing optimal male antennae construction, which we plan to look at heterogeneity in male moth quality through their movement strategies. The other is employing interruption of pheromone signaling, which could be used as a population control mechanism as an alternative to the use of pesticides. Using the model as a “sandbox,” it will be possible to devise population control strategies.